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Understanding Probabilities and probability for single and conditional events

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Introduction : One conception of probability is drawn from the idea of symmetrical outcomes. For example, the two possible outcomes of tossing a fair coin seem not to be distinguishable in any way that affects which side will land up or down. Therefore the probability of heads is taken to be 1/2, as is the probability of tails. In general, if there are N symmetrical outcomes, the probability of any given one of them occurring is taken to be 1/N. Thus, if a six-sided die is rolled, the probability of any one of the six sides coming up is 1/6.

Probabilities can also be thought of in terms of relative frequencies. If we tossed a coin millions of times, we would expect the proportion of tosses that came up heads to be pretty close to 1/2. As the number of tosses increases, the proportion of heads approaches 1/2. Therefore, we can say that the probability of a head is 1/2.

Probability of a Single Event

If you roll a six-sided die, there are six possible outcomes, and each of these outcomes is equally likely. A six is as likely to come up as a three, and likewise for the other four sides of the die. What, then, is the probability that a one will come up? Since there are six possible outcomes, the probability is 1/6. What is the probability that either a one or a six will come up? The two outcomes about which we are concerned (a one or a six coming up) are called favorable outcomes. Given that all outcomes are equally likely, we can compute the probability of a one or a six using the formula:

$$\text{probability} = \frac{\text{Number of favorable outcomes}}{\text{Number of possible equally-likely outcomes}}$$

In this case there are two favorable outcomes and six possible outcomes. So the probability of throwing either a one or six is 1/3. Don't be misled by our use of the term "favorable," by the way. You should understand it in the sense of "favorable to the event in question happening." That event might not be favorable to your well-being. You might be betting on a three, for example.

The above formula applies to many games of chance. For example, what is the probability that a card drawn at random from a deck of playing cards will be an ace? Since the deck has four aces, there are four favorable outcomes; since the deck has 52 cards, there are 52 possible outcomes. The probability is therefore 4/52 = 1/13. What about the probability that the card will be a club? Since there are 13 clubs, the probability is 13/52 = 1/4.

Let's say you have a bag with 20 cherries: 14 sweet and 6 sour. If you pick a cherry at random, what is the probability that it will be sweet? There are 20 possible cherries that could be picked, so the number of possible outcomes is 20. Of these 20 possible outcomes, 14 are favorable (sweet), so the probability that the cherry will be sweet is 14/20 = 7/10. There is one potential complication to this example, however. It must be assumed that the probability of picking any of the cherries is the same as the probability of picking any other. This wouldn't be true if (let us imagine) the sweet cherries are smaller than the sour ones. (The sour cherries would come to hand more readily when you sampled from the bag.) Let us keep in mind, therefore, that when we assess probabilities in terms of the ratio of favorable to all potential cases, we rely heavily on the assumption of equal probability for all outcomes.

Probability of Two (or more) Independent Events

Events A and B are independent events if the probability of Event B occurring is the same whether or not Event A occurs. Let's take a simple example. A fair coin is tossed two times. The probability that a head comes up on the second toss is 1/2 regardless of whether or not a head came up on the first toss. The two events are (1) first toss is a head and (2) second toss is a head. So these events are independent. Consider the two events (1) "It will rain tomorrow in Houston" and (2) "It will rain tomorrow in Galveston" (a city near Houston). These events are not independent because it is more likely that it will rain in Galveston on days it rains in Houston than on days it does not.

Conditional Probabilities

Often it is required to compute the probability of an event given that another event has occurred. For example, what is the probability that two cards drawn at random from a deck of playing cards will both be aces? It might seem that you could use the formula for the probability of two independent events and simply multiply 4/52 x 4/52 = 1/169. This would be incorrect, however, because the two events are not independent. If the first card drawn is an ace, then the probability that the second card is also an ace would be lower because there would only be three aces left in the deck.

Once the first card chosen is an ace, the probability that the second card chosen is also an ace is called the conditional probability of drawing an ace. In this case, the "condition" is that the first card is an ace. Symbolically, we write this as:

P(ace on second draw | an ace on the first draw)

The vertical bar "|" is read as "given," so the above expression is short for: "The probability that an ace is drawn on the second draw given that an ace was drawn on the first draw." What is this probability? Since after an ace is drawn on the first draw, there are 3 aces out of 51 total cards left. This means that the probability that one of these aces will be drawn is 3/51 = 1/17.

If Events A and B are not independent, then P(A and B) = P(A) x P(B|A).

Applying this to the problem of two aces, the probability of drawing two aces from a deck is 4/52 x 3/51 = 1/221.

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